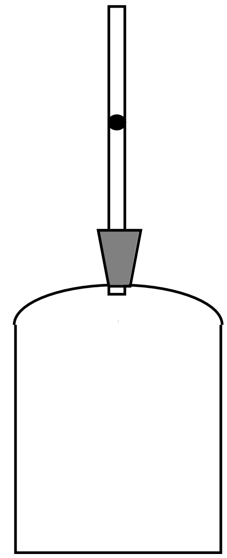
SP226 **Measurement of C_p/C_v for Air** Spring 2000

Purpose: To measure the ratio of the specific heat at constant pressure to the specific heat at constant volume ($\gamma = C_p/C_v$) for air and compare your result to the prediction of kinetic theory.



Method: The ratio $\gamma = C_p/C_v$ is used to describe processes in which a gas undergoes adiabatic compressions or expansions. You can measure γ by measuring the speed of sound in the gas (the compressions and expansions are so rapid that they are adiabatic) or by measuring the properties of other rapid oscillatory motions of the gas.

In this experiment, you will drop a small metal ball through a closely-fitting glass tube that is connected, through a rubber stopper, to a large glass bell jar. This apparatus is illustrated in the figure. As the ball falls, it will compress the air trapped inside the jar, raising its pressure. This will eventually stop the ball and cause it to turn around: that is, the ball will "bounce" on the air trapped inside the jar. As the ball rises, the air in the jar will expand, its pressure will decrease, and the ball will slow down again. Once the ball stops rising, it will begin to fall, and the process will repeat itself.

If there were no friction between the ball and the tube, the ball would execute simple harmonic motion, oscillating forever. In fact, it executes damped harmonic motion, but we will ignore the damping.

The change in pressure as the volume of the trapped air changes determines the ball's oscillation period. The relation between pressure and volume depends on γ, so by measuring the period, a value for γ can be deduced.

Reference: Tipler, Chapter 19, Section 6, pp. 578-583.

Theory: If you apply Newton's Second Law to the ball at its equilibrium position, you will find that the upward force exerted by the air in the jar just balances the downward force of the air outside plus the weight:

$$P_{\text{eq}} A - P_{\text{atm}} A - mg = 0 . \qquad \text{(eq. 1)}$$

where P_{eq} is the equilibrium pressure in the jar. If you now displace the ball downward, the pressure will increase by an amount $\Delta P = P - P_{\text{eq}}$, and the net force on the ball must equal $A\Delta P$. In accord with Newton's second law, this force equals the product of the ball's mass and its acceleration:

$$A\Delta P = m \frac{d^2 y}{dt^2} . \qquad \text{(eq. 2)}$$

To relate the change in pressure to the ball's displacement, recall that, for an adiabatic process,

$$PV^{\gamma} = \text{constant}$$

so

$$d(PV^{\gamma}) = P\gamma V^{\gamma-1} dV + V^{\gamma} dP = 0$$

Replacing differentials with finite (but small) differences and rearranging gives

$$\Delta P = -P\gamma \frac{\Delta V}{V} . \qquad \text{(eq. 3)}$$

Note: If the compressions were isothermal, instead of adiabatic, you'd get a different relationship.

If the displacement of the ball from its equilibrium position is denoted by *y*, then the change in volume of the trapped air is $\Delta V = Ay$, where *A* is the ball's cross sectional area. The change in pressure is then given by

$$\Delta P = -P\gamma \frac{\Delta V}{V} = -P\gamma \frac{Ay}{V}. \qquad \text{(eq. 4)}$$

Substituting this back into (eq. 2) gives

$$-P\gamma \frac{A^2}{V} y = m \frac{d^2 y}{dt^2},$$

which we can write in the more standard form

$$\frac{d^2 y}{dt^2} + \omega^2 y = 0 \qquad \text{(eq. 5)}$$

where

$$\omega = A\sqrt{\frac{P\gamma}{mV}}$$

This equation describes simple harmonic motion, with period

$$T = \frac{2\pi}{\omega} = \frac{8}{d^2}\sqrt{\frac{mV}{P\gamma}} \qquad \text{(eq. 6)}$$

where *A* was replaced by $\pi d^2/4$. If you measure *T* for known values of *m*, *V*, *A*, and *P*, where *P* is the equilibrium pressure, you can calculate γ. Solving eq. 6 for γ gives

$$\gamma = 64\frac{mV}{Pd^4} \qquad \text{(eq. 7)}$$

Procedure:

1. Look over your apparatus.

Measure and record the mass *m* and diameter *d* of the ball. Calculate the ball's cross-sectional area *A*. Calculate the pressure P_{eq} inside the jar when the ball is at its equilibrium position. Assume that atmospheric pressure is $p_{atm} = 1.01 \times 10^5$ Pa.

Calculate the volume *V* of the bell jar. The nominal volume is *V* = 10 L, and you may use this value in your analysis. You should convince yourself, however, that this value is about right. Measure the relevant dimensions and show your calculation.

Start the LabView instrument FLAG.TIMER5. Set BLOCKED (SEC) and UNBLOCKED (SEC) to the range 0.0000<T<6.5000.

2. Measure the ball's period of oscillation.

Be sure that the rubber stopper with the stopcock is firmly inserted in the port at the bottom of the bell-jar and that the stopcock is closed (so that the air is trapped in the jar).

Place the photogate so it can "see" the ball when it passes through the glass tube. Press INITIALIZE! on the FLAG.TIMER5 control screen. Then press START READING.

Insert the ball gently into the glass tube — don't force it or you may crack the tube. Let the ball fall and bounce until it stops. [Note: This is the hardest part of the experiment, and you'll need to exercise some patience and finesse to get good data. If the ball seems too tight in the tube, maybe it has warmed up and expanded as you handled it. Let it cool. Or maybe it's got some dust on it. Wipe it off with a lint-free wiper.

Keep trying. If you get frustrated, ask for help.] Press STOP READING and pass your fingers through the photogate a few times to get the software to display the times it has measured.

Examine the column labeled ALT. FLG. INT. That's where the oscillation period will be. Ignore the first and last readings, and copy down the others on a piece of paper. Maybe you'll only get one or two per trial if the damping is large.

Repeat until you have "enough" measurements of the period — at least 20. Then calculate the mean value and standard deviation.

3. Calculate γ for air.

Use (eq. 6) to find a value for γ. Estimate its uncertainty from the uncertainties in the various measured quantities.

4. Repeat your measurements for argon gas.

If you have time, fill the bell jar with argon gas from the bottle in the lab room. Argon is inert, so it won't hurt you, and it's heavier than air, so it will (mostly) stay in the bell jar. Don't fill the room with argon, though, because you will asphyxiate.

Measure the period of oscillation and calculate a value of γ for argon.

Conclusions: Compare your measured values of γ for air and for argon with those predicted by kinetic theory. Discuss the agreement.

Report: Submit your data for m, A, and V, and a data table containing your values of T for each gas. Submit a sample calculation of γ for each gas and a discussion of the agreement between your measured values and the ones you expected.